A New Class of Parameter Shapes for Generalized iLUCK Models

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Generalized iLUCK models

- a way to efficiently describe imprecise Bayesian inference by using sets of natural conjugate priors
- sets of priors defined via sets of canonical (hyper)parameters $n^{(0)}$, $y^{(0)}$
  - $n^{(0)}$: pseudocounts, prior strength
  - $y^{(0)}$: prior guess on parameter of interest from sampling distr.

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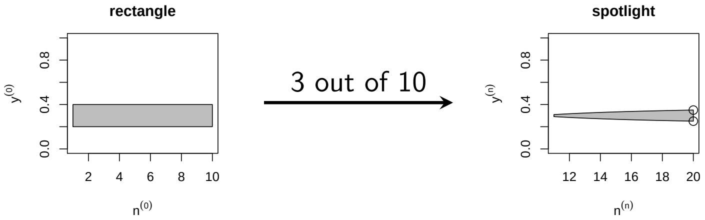
Imprecise Beta-Bernoulli/Binomial Model (IBBM)

Data :	s	$\sim$	$\text{Binom}(p, n)$
conjugate prior:	p	$\sim$	$\text{Beta}(n^{(0)}, y^{(0)})$
posterior:	$p \mid s$	$\sim$	$\text{Beta}(n^{(n)}, y^{(n)})$

$$y^{(n)} = \frac{n^{(0)}}{n^{(0)} + n} \cdot y^{(0)} + \frac{n}{n^{(0)} + n} \cdot \frac{s}{n}, \qquad n^{(n)} = n^{(0)} + n$$

Prior-Data Conflict and Set Shapes

- systematic reaction to prior-data conflict only with prior sets where $n^{(0)}$ varies.
- set shape has influence on prior-data conflict:



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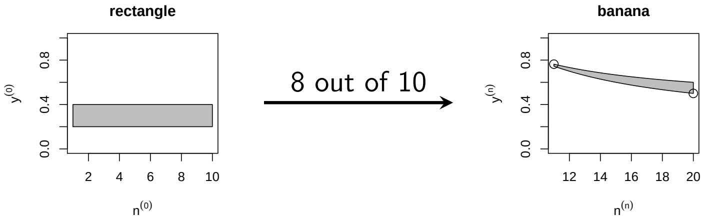

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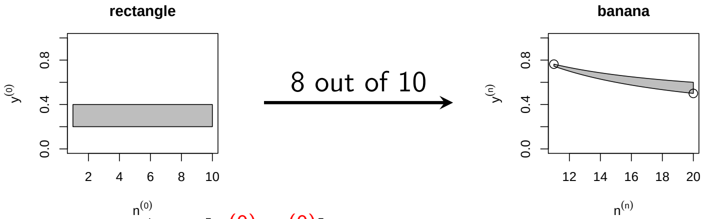

- when $s/n \notin [\underline{y}^{(0)}, \overline{y}^{(0)}]$, then additional imprecision.
- rectangle prior set is just first step, any shape is possible.
- take lower and upper borders as functions of $n^{(0)}$: $\underline{y}^{(0)}(n^{(0)})$ / $\overline{y}^{(0)}(n^{(0)})$, where $n^{(0)} \in [\underline{n}^{(0)}, \overline{n}^{(0)}]$
- define these functions by positions at left and right endpoints and a parameter to characterize the function in between

Precision of Elicitation Perspective

- for low $n^{(0)}$, can be precise with $y^{(0)}$ ➡ short $y^{(0)}$ interval
- for high $n^{(0)}$, must be cautious with $y^{(0)}$ ➡ long $y^{(0)}$ interval

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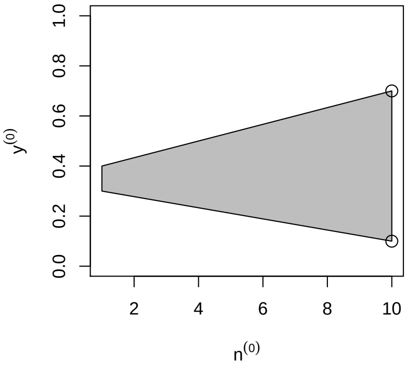

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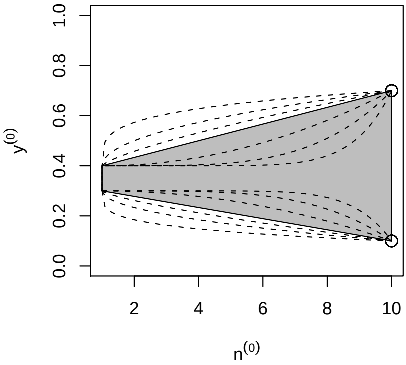
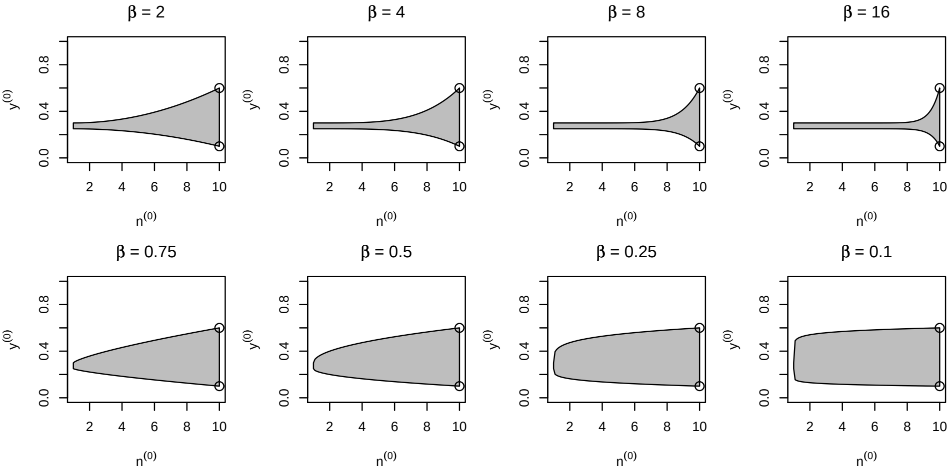


$$\overline{y}^{(0)}(n^{(0)})$$

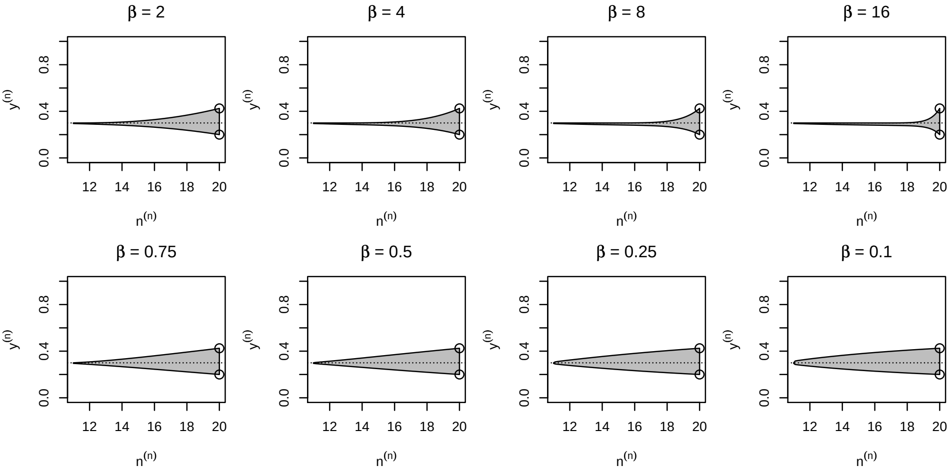
$$= \overline{y}_L^{(0)} + \left[\overline{y}_R^{(0)} - \overline{y}_L^{(0)}\right] \underbrace{\left(\frac{n^{(0)} - \underline{n}^{(0)}}{\overline{n}^{(0)} - \underline{n}^{(0)}}\right)}_{=:\,\tilde{n}^{(0)}}{}^{\overline{\beta}}$$

$$= \overline{y}_R^{(0)} \left[\tilde{n}^{(0)}\right]^{\overline{\beta}} + \overline{y}_L^{(0)} \left(1 - \left[\tilde{n}^{(0)}\right]^{\overline{\beta}}\right)$$

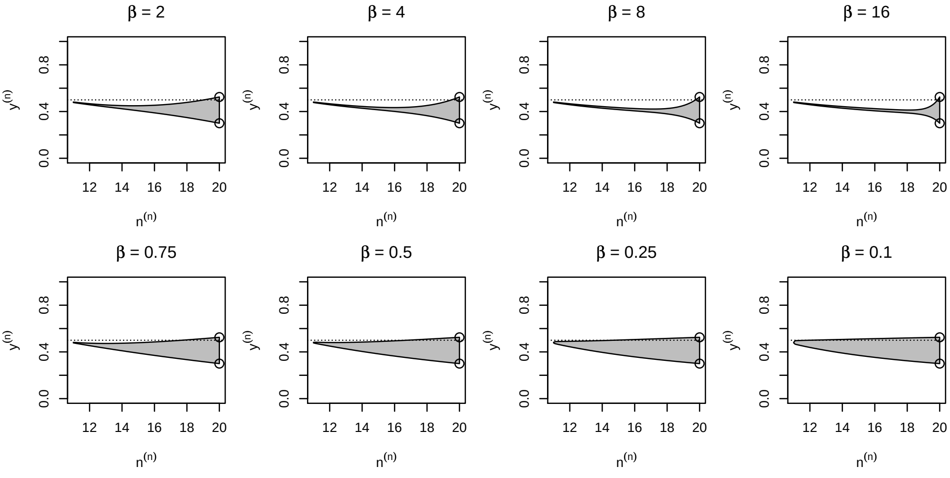
Precision of Elicitation Perspective: Priors



Posteriors when $s/n = 3/10$



Posteriors when $s/n = 5/10$



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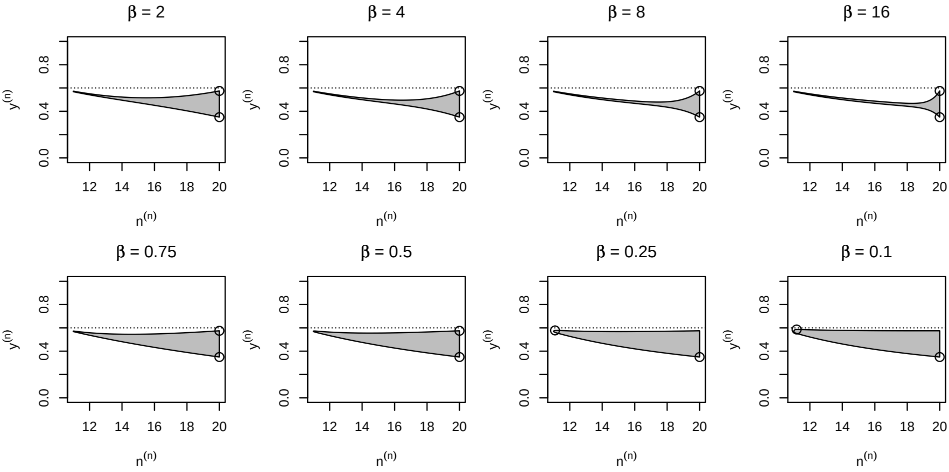
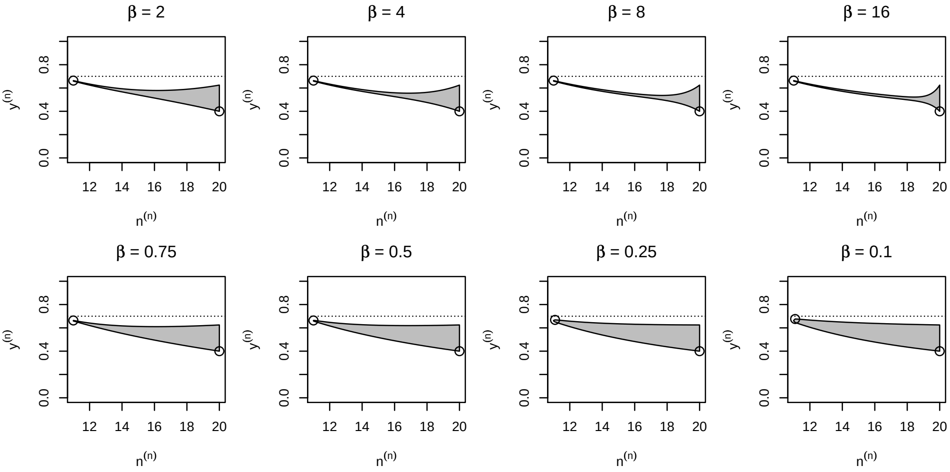
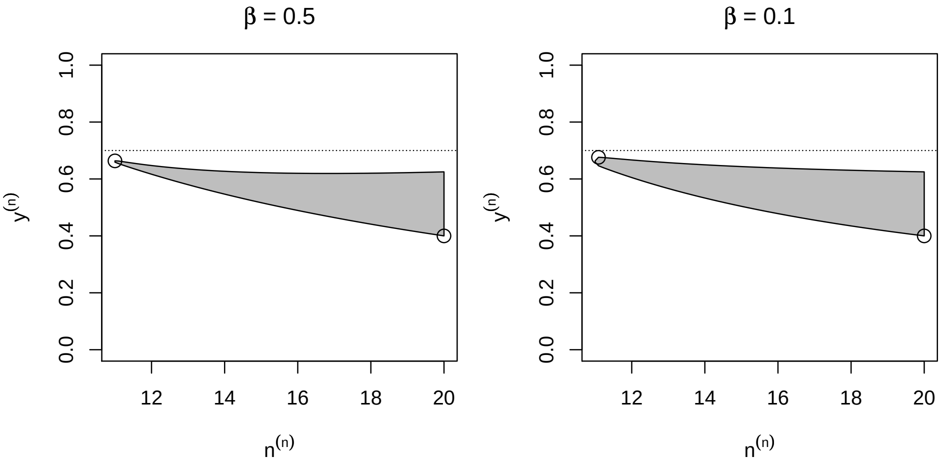


Posteriors when $s/n = 7/10$



Posteriors when $s/n = 7/10$ — Zoom on $\beta = 0.5, 0.1$



Precision of Elicitation Perspective

- Sets with $\beta > 1$ show nice 'tolerance' behaviour: additional imprecision only when 'spike' at $\underline{n}^{(n)}$ overtakes $\overline{y}^{(n)}(\overline{n}^{(n)})$
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Velocity of Precision Increase Perspective

- $y^{(0)}$ values at low $n^{(0)}$ move faster, so $y^{(0)}$ interval shrinks faster and thus can be larger at the beginning.
- $y^{(0)}$ values at high $n^{(0)}$ move slower, so $y^{(0)}$ interval shrinks slower and thus should be smaller at the beginning so it doesn't dominate the inferences.

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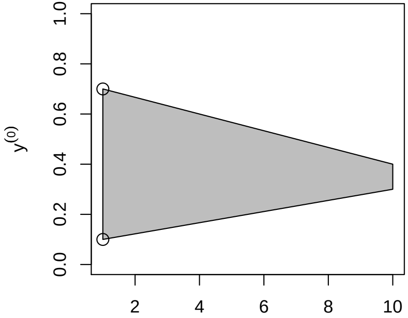

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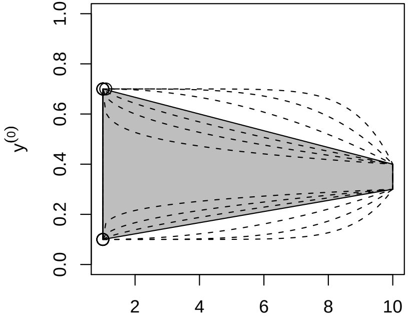

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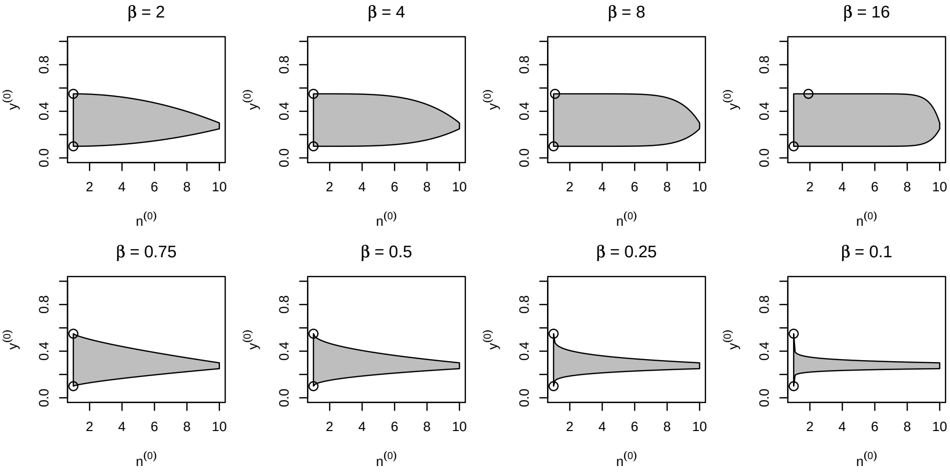

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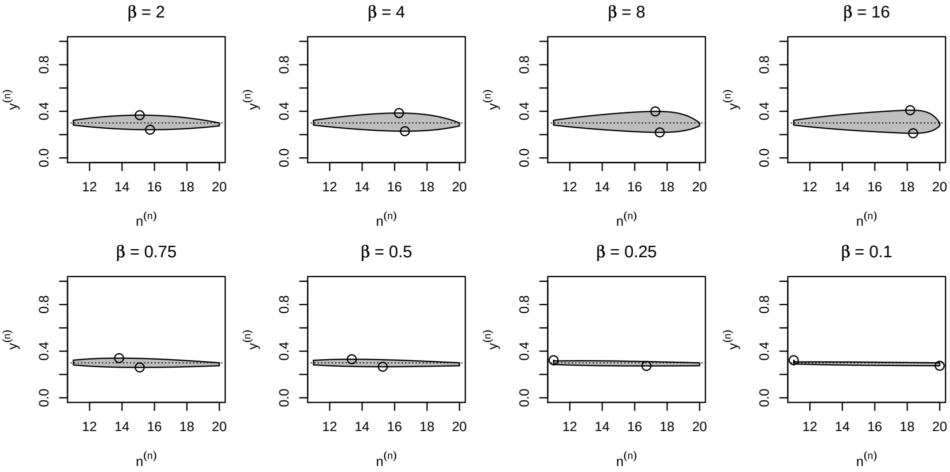

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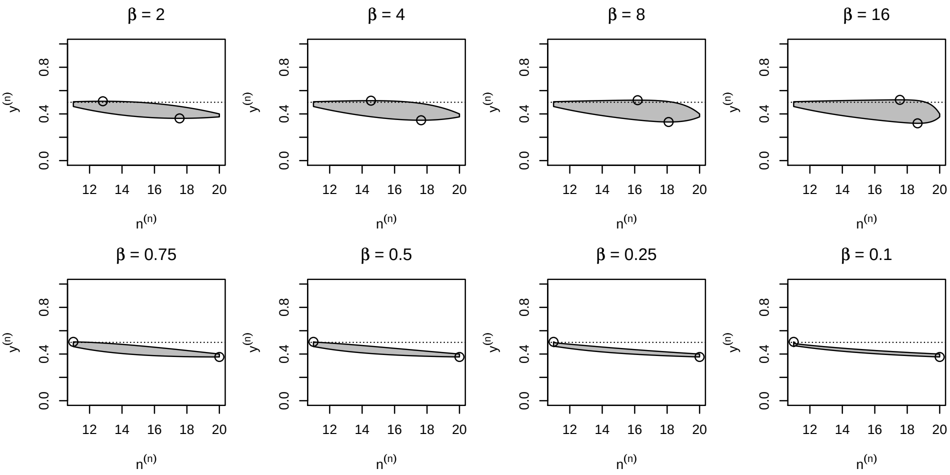

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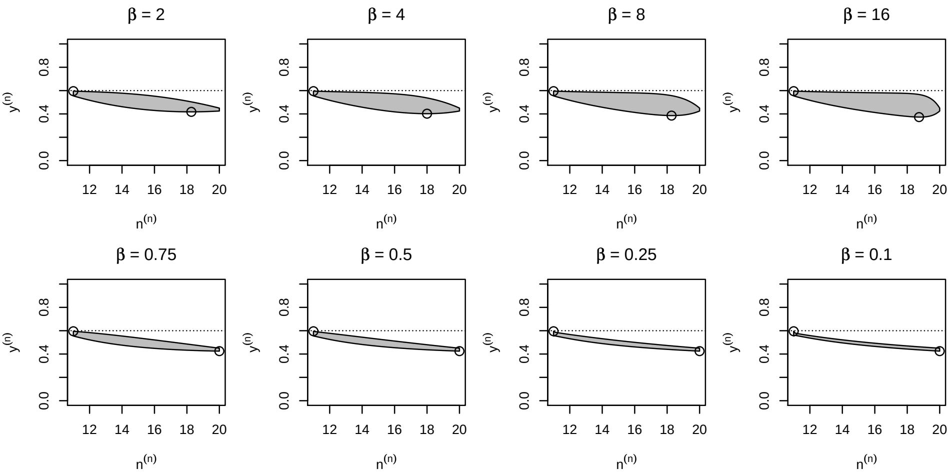

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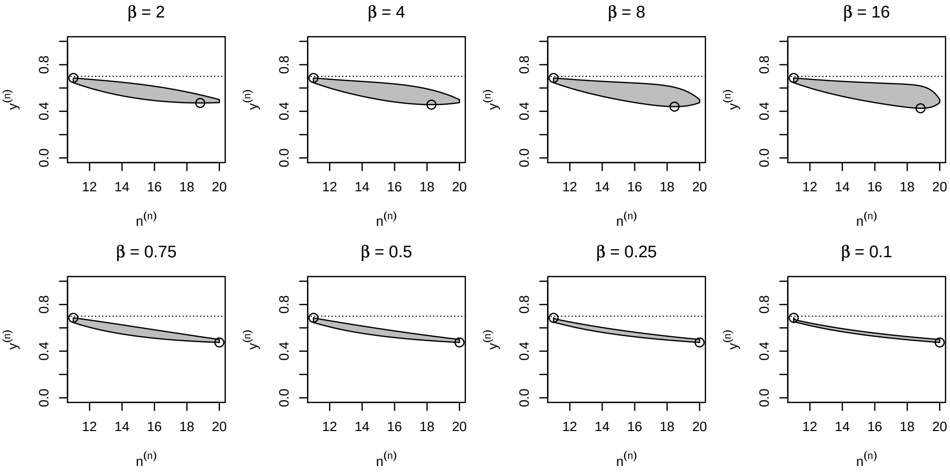
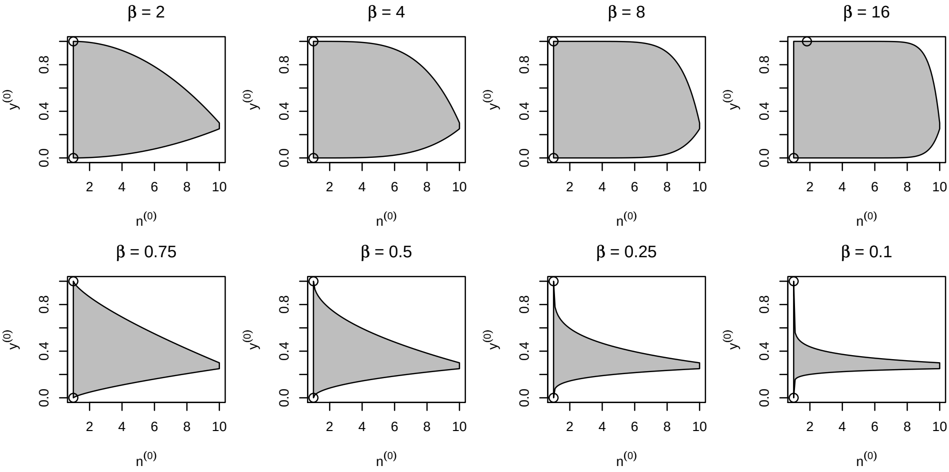


Velocity of Precision Increase Perspective

- for 'non-conflicting' observations, $\underline{y}^{(n)}$ and $\overline{y}^{(n)}$ are attained neither at $\underline{n}^{(0)}$ or $\overline{n}^{(0)}$.
- for 'conflicting' observations, nearest $y^{(n)}$ attained at $\underline{n}^{(0)}$ as for rectangle set. ($\beta > 1$: Farest $y^{(n)}$ may stay away from $\underline{y}^{(n)}$ and $\overline{y}^{(n)}$.)
- Therfore no easily interpretable updating rules!
- 'picky' for $\beta < 1$, 'tolerance' reaction for $\beta > 1$.
- Does $[\underline{y}_L^{(0)}, \overline{y}_L^{(0)}] = [0, 1]$ makes sense?
  (Would make elicitaton easier.)

Velocity of Precision Increase Perspective: Priors



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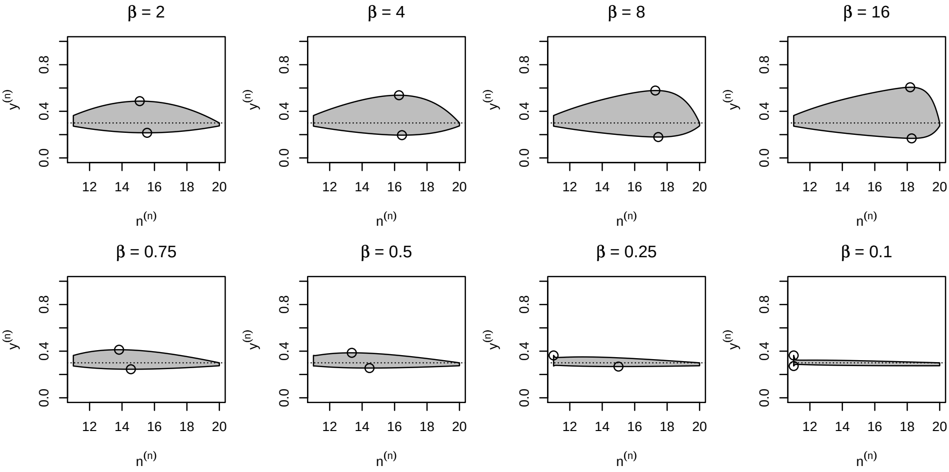

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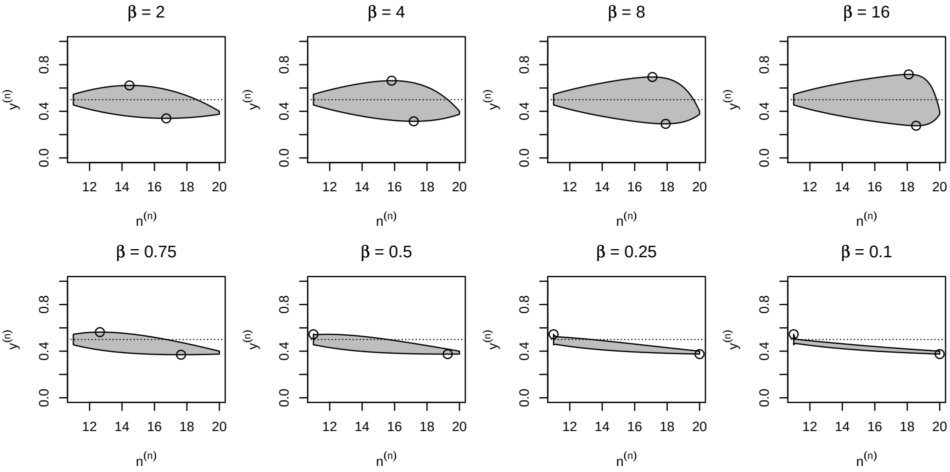

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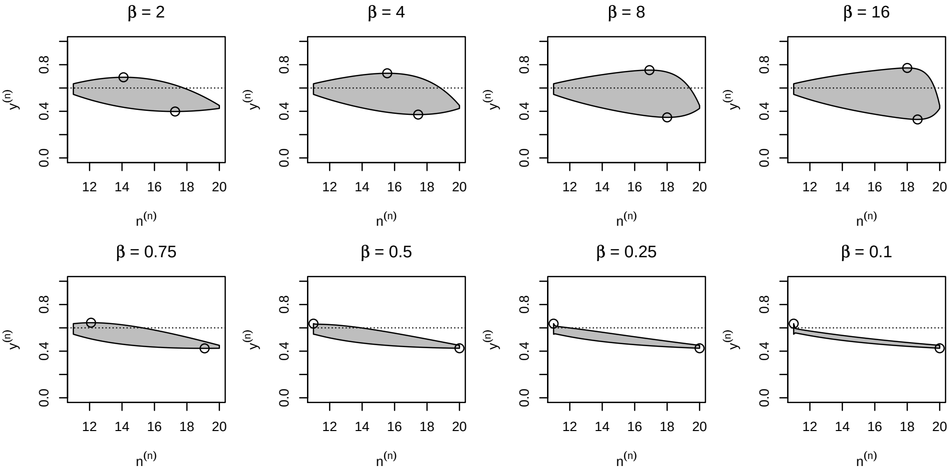

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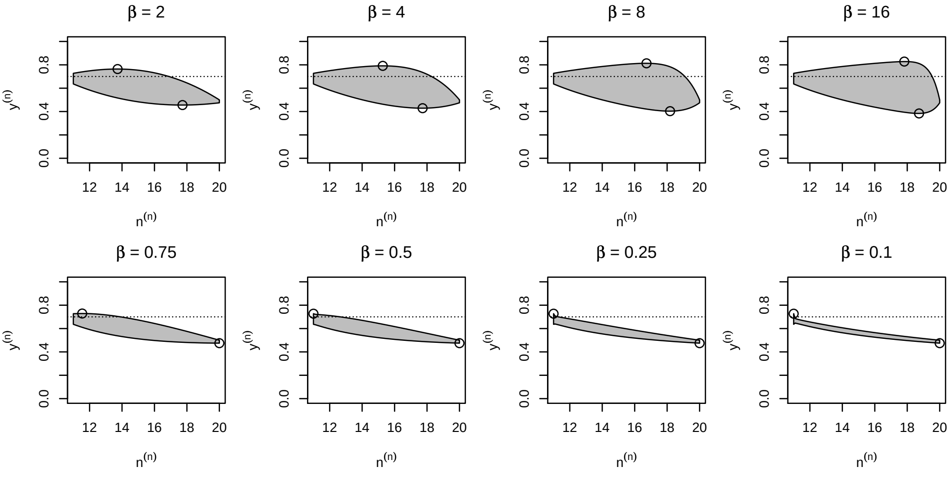

Further Questions

- Is there a shape description that is invariant to updating? (other than general subset, or with fixed $n^{(0)}$.)
- Is there a transformation of this parameter space that makes it easier to see what's going on?
- Look at Predictive Probability Plots (PPPs)
- ...