Nonparametric System Reliability Combining Expert Knowledge and Data

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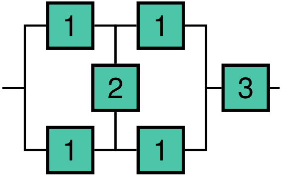
System Reliability



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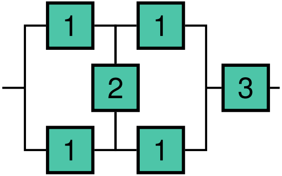

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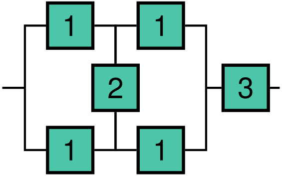

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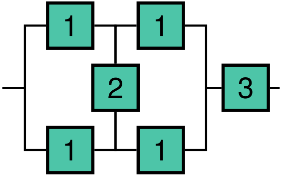

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How to combine these two information sources?

Bayesian Inference

expert info + data → complete picture

Bayesian Inference

expert info	+	data	$\rightarrow$	complete picture
prior distribution	+	sample distribution	$\rightarrow$	posterior distribution
$f(p)$	$\times$	$f(s \mid p)$	$\propto$	$f(p \mid s)$

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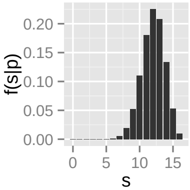

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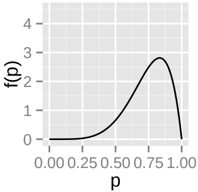

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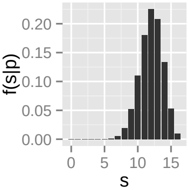

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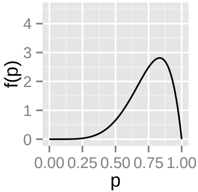

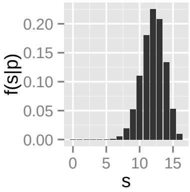

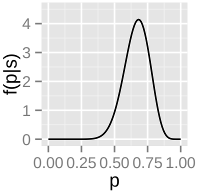

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- conjugate prior makes learning about parameter tractable, just update hyperparameters: $\alpha^{(0)} \to \alpha^{(n)}$, $\beta^{(0)} \to \beta^{(n)}$
- closed form for some inferences: $\text{E}[p \mid s] = \frac{\alpha^{(n)}}{\alpha^{(n)}+\beta^{(n)}}$

Nonparametric Component Reliability

Functioning probability p_t^k of k for each time $t \in \mathcal{T} = \{t_1, t_2, \dots\}$

- discrete component reliability function $R^k(t) = p_t^k,\ t \in \mathcal{T}$.

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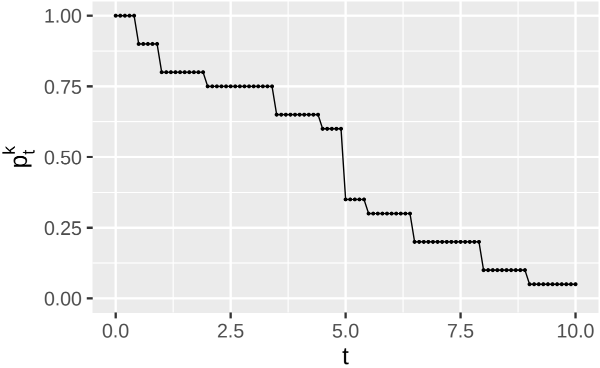

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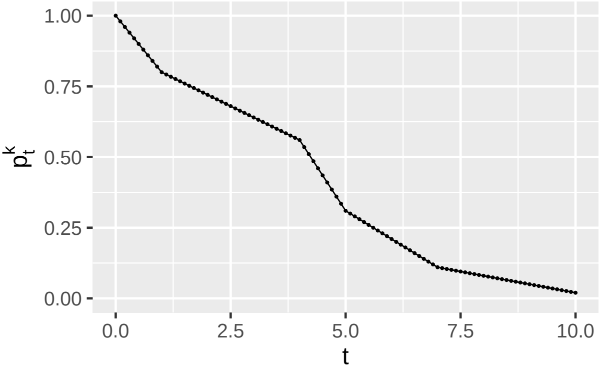

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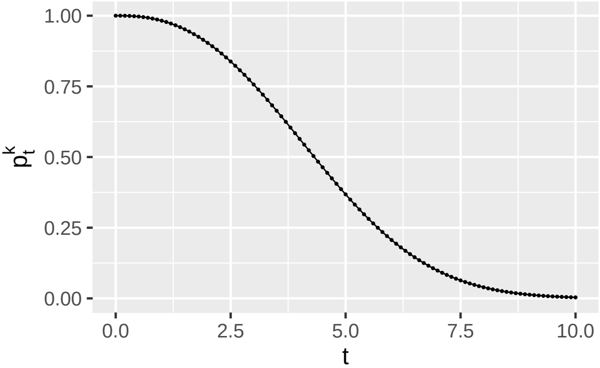

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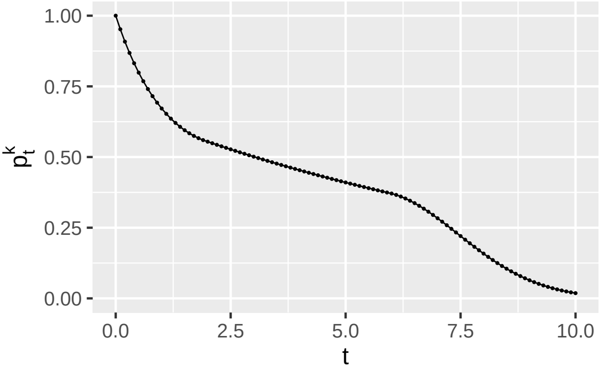

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  $$p_t^k \sim \text{Beta}(n_{k,t}^{(0)}, y_{k,t}^{(0)}) \qquad n_{k,t}^{(0)} = \alpha_{k,t}^{(0)} + \beta_{k,t}^{(0)}, \quad y_{k,t}^{(0)} = \frac{\alpha_{k,t}^{(0)}}{\alpha_{k,t}^{(0)} + \beta_{k,t}^{(0)}} = \text{E}[p_t^k]$$

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System Reliability

- Closed form for the system reliability via the survival signature:

$$P\left(T_{\mathsf{sys}} > t \mid \{n_{k,t}^{(0)}, y_{k,t}^{(0)}, \boldsymbol{t}^k\}^{1:K}\right) = \sum_{l_1=0}^{m_1} \cdots \sum_{l_K=0}^{m_K} \Phi(l_1, \ldots, l_K) \prod_{k=1}^{K} P(C_t^k = l_k \mid n_{k,t}^{(0)}, y_{k,t}^{(0)}, \boldsymbol{t}^k)$$

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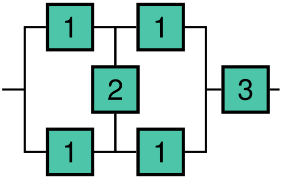

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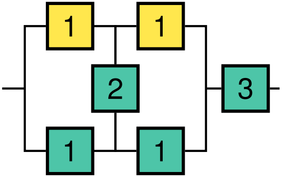

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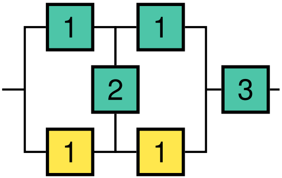

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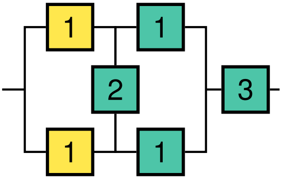

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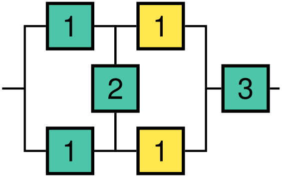

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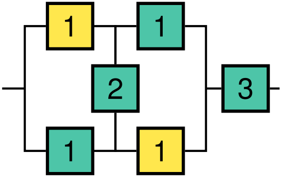

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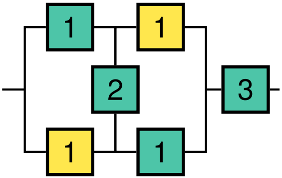

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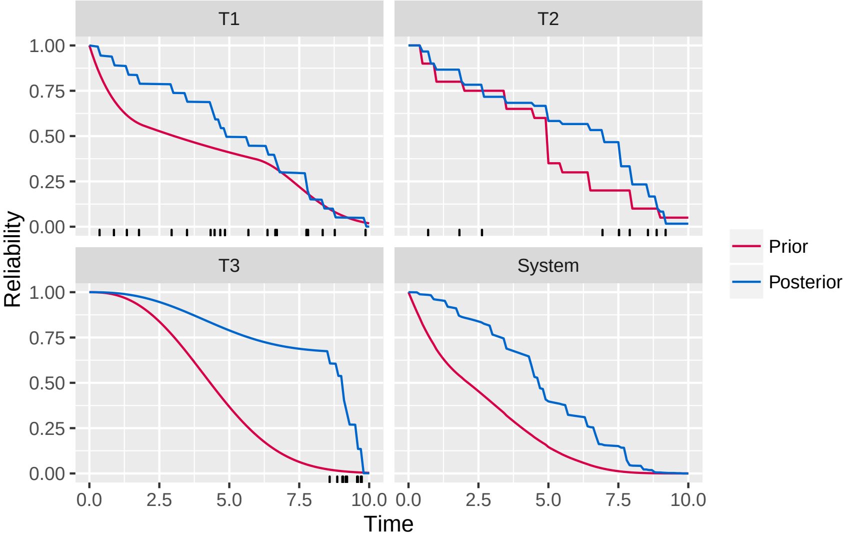
Posterior predictive probability that in a new system, l_k of the m_k k 's function at time t:

$$\binom{m_k}{l_k} \int [P(T < t \mid p_t^k)]^{l_k} [P(T \geq t \mid p_t^k)]^{m_k - l_k} f(p_t^k \mid n_{k,t}^{(0)}, y_{k,t}^{(0)}, \boldsymbol{t}^k)\, dp_t^k$$

(integral can be solved analytically)

System Reliability: Example



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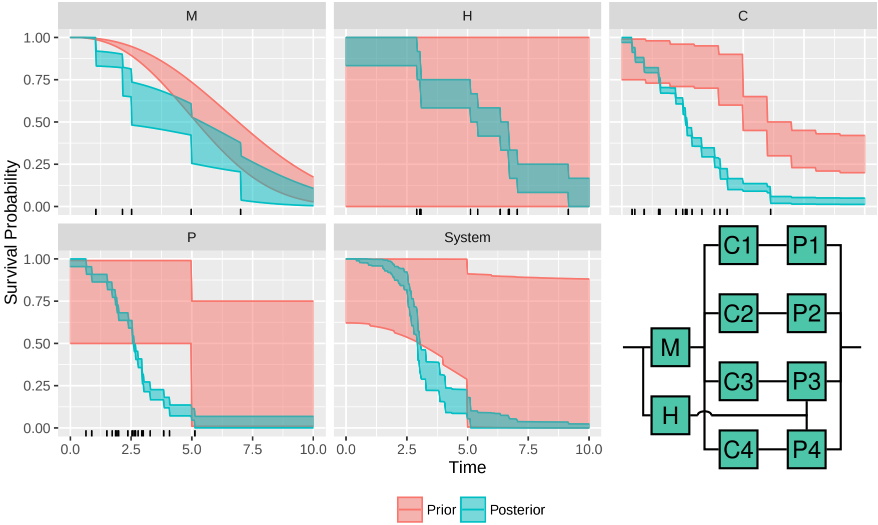
  - min and max $R_{\text{sys}}(t)$ over $\Pi^{(0)}$ analytical in most cases!

System Reliability Bounds



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Next steps:

- Allow right-censored observations (RUL estimation)
- Allow dependence between components (common-cause failure, . . .)
- Use for system design (where to put extra redundancy?)
- Use for maintenance planning

References

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