Nonparametric Bayesian System Reliability using Sets of Priors

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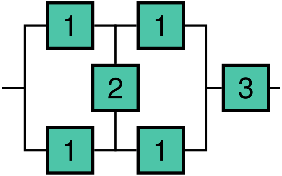
University of Oxford

Durham University

DINALOG Dutch Institute for Advanced Logistics

Hannover 2016-12-08

System Reliability



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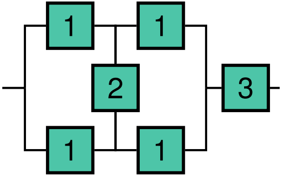

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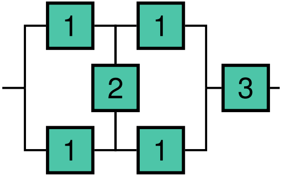

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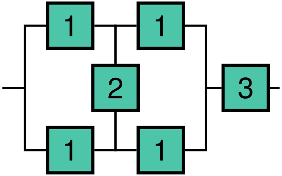

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How to combine these two information sources?

Bayesian Inference

expert info + data → complete picture

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expert info	+	data	→	complete picture
prior distribution	+	sample distribution	→	posterior distribution
$f(p)$	$\times$	$f(s \mid p)$	$\propto$	$f(p \mid s)$

► Bayes' Rule

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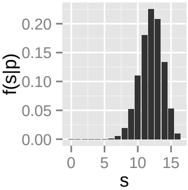
Binomial distribution

$$s \mid p \sim \text{Binomial}(n, p)$$



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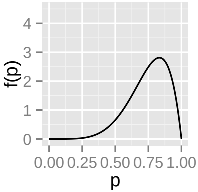

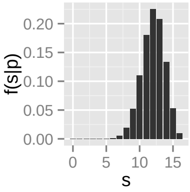

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				► conjugacy
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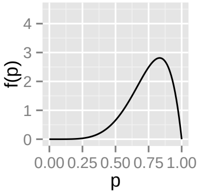

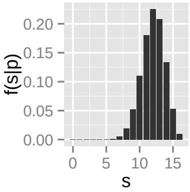

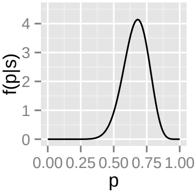

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- conjugate prior makes learning about parameter tractable, just update hyperparameters: $\alpha^{(0)} \rightarrow \alpha^{(n)}$, $\beta^{(0)} \rightarrow \beta^{(n)}$
- closed form for many inferences, e.g. $\text{E}[p \mid s] = \frac{\alpha^{(n)}}{\alpha^{(n)}+\beta^{(n)}}$

Nonparametric Component Reliability

Functioning probability p_t^k of k for each time $t \in \mathcal{T} = \{t'_1, t'_2, \ldots\}$

- discrete component reliability function $R^k(t) = p_t^k,\ t \in \mathcal{T}$.

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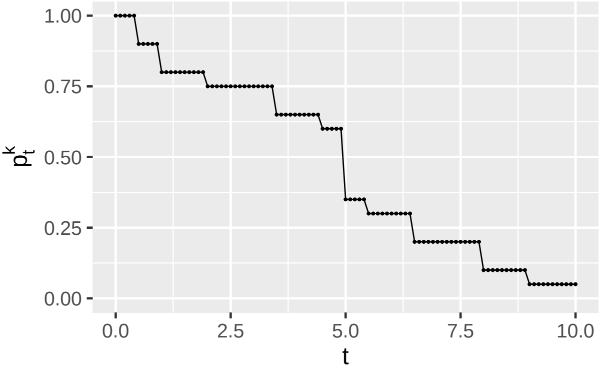

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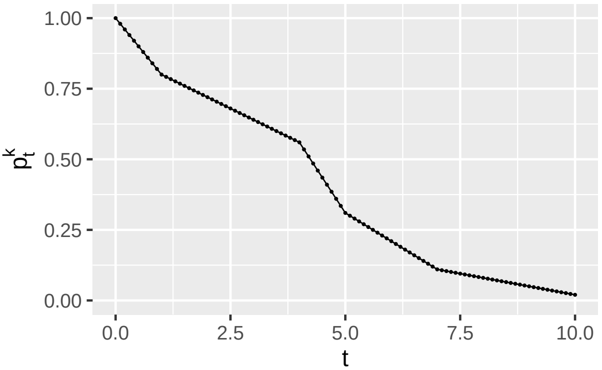

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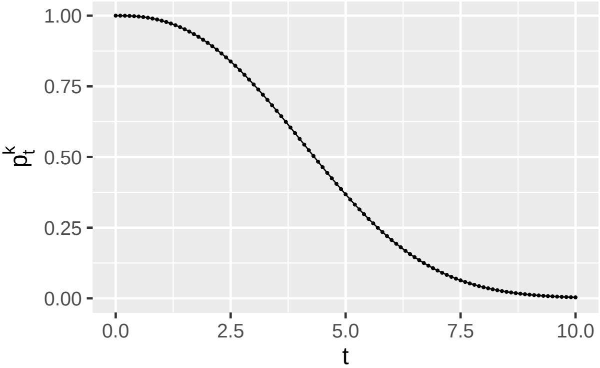

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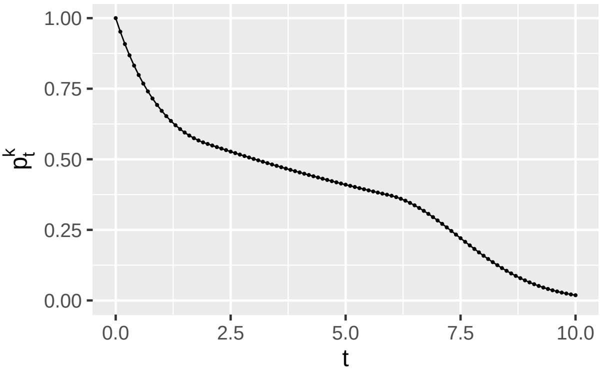

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Prior-Data Conflict

What if expert information and data tell different stories?

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- *informative prior beliefs* and *trusted data* (sampling model correct, no outliers, etc.) are in conflict
- "[...] the prior [places] its mass primarily on distributions in the sampling model for which the observed data is surprising" (Evans and Moshonov 2006)
- there are not enough data to overrule the prior

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$$\mathrm{Var}[p \mid s] = \frac{y^{(n)}(1 - y^{(n)})}{n^{(n)} + 1}$$ decreases with n!

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Uncertainty about probability statements

smaller sets = more precise probability statements

Lottery A
Number of winning tickets:
exactly known as 5 out of 100
▶ $P(\text{win}) = 5/100$

Lottery B
Number of winning tickets:
not exactly known, supposedly
between 1 and 7 out of 100
▶ $P(\text{win}) = [1/100, 7/100]$

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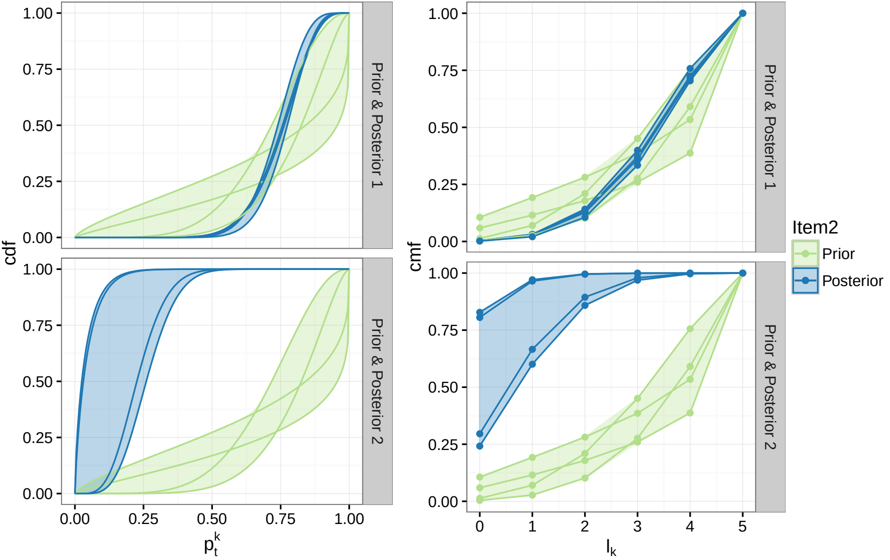
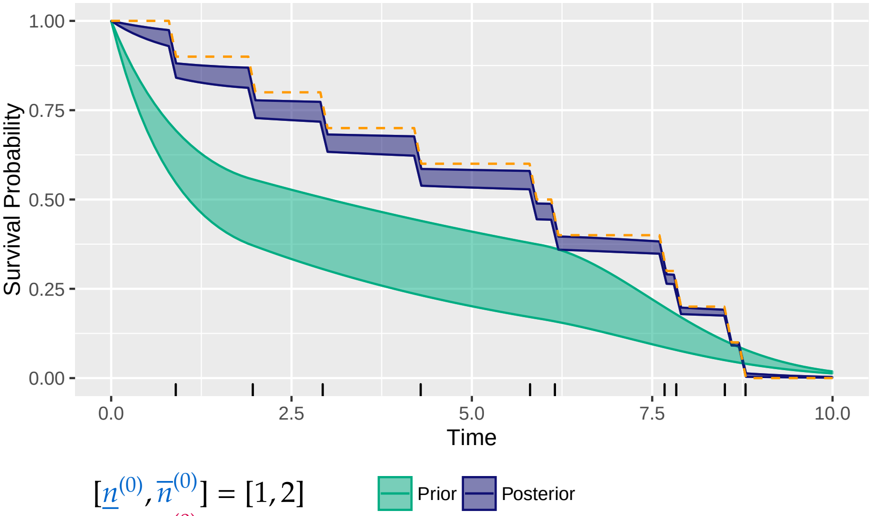
- Bounds for inferences (point estimate, prediction, . . .) by min/max over

Sets of Priors for p_t^k and C_t^k



Component Reliability with Sets of Priors



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$[\underline{y}^{(0)}, \overline{y}^{(0)}] = \updownarrow$

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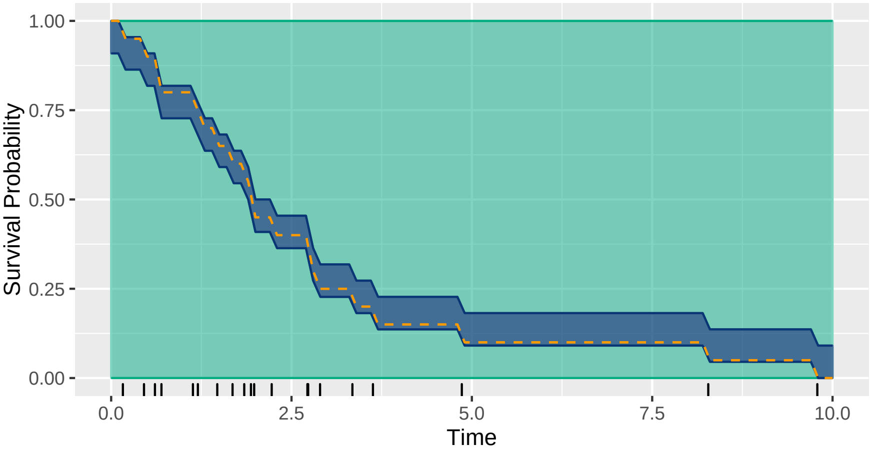

$[\underline{n}^{(0)}, \overline{n}^{(0)}] = [1, 2]$

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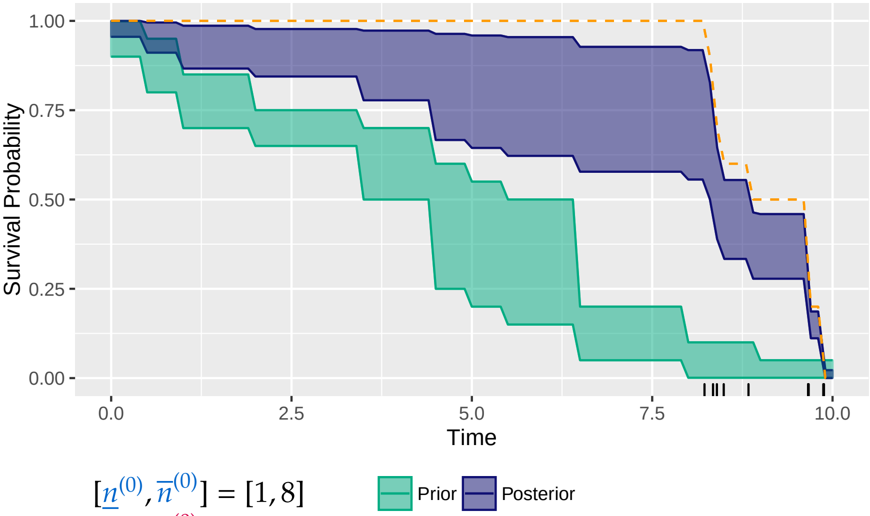

$[\underline{n}^{(0)}, \overline{n}^{(0)}] = [1, 8]$

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- Closed form for the system reliability via the survival signature:

$$R_{\text{sys}}\left(t \mid \left\{n_{k,t}^{(0)}, y_{k,t}^{(0)}, \boldsymbol{t}^k\right\}^{k=1:K}\right) = P(T_{\text{sys}} > t \mid \cdots)$$

$$= \sum_{l_1=0}^{m_1} \cdots \sum_{l_K=0}^{m_K} \Phi(l_1, \ldots, l_K) \prod_{k=1}^{K} P(C_t^k = l_k \mid n_{k,t}^{(0)}, y_{k,t}^{(0)}, \boldsymbol{t}^k)$$

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Survival signature $\Phi(l_1, \ldots, l_K)$
(Coolen and Coolen-Maturi 2012)
$= P($system functions $\mid \{l_k$ k 's function$\}^{1:K})$

l_1	l_2	l_3	Φ
0	0	1	0
1	0	1	0
2	0	1	1/3
3	0	1	1
4	0	1	1

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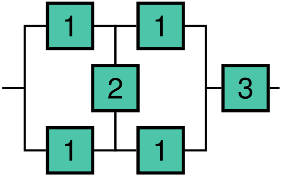

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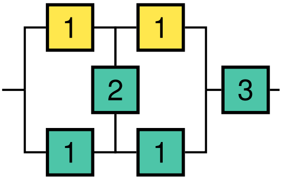
✓



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$= P(\text{system functions} \mid \{l_k$ k 's function$\}^{1:K})$

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0	0	1	0
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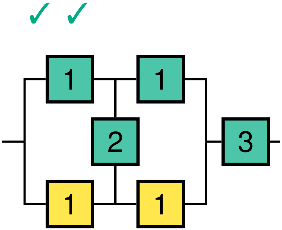

System Reliability

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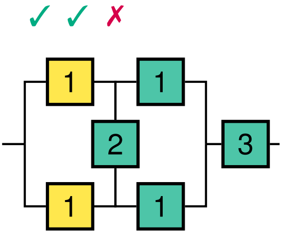

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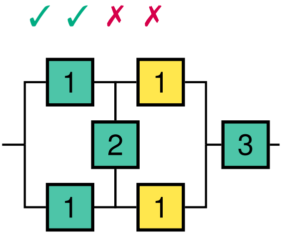

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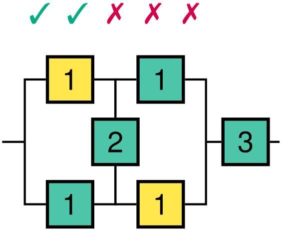

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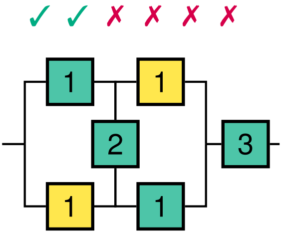

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Posterior predictive probability that in a new system, l_k of the m_k k 's function at time t:

$$\binom{m_k}{l_k} \int_0^1 \big[p^k_t\big]^{l_k} \big[1 - p^k_t\big]^{m_k - l_k} \times f(p^k_t \mid n^{(0)}_{k,t}, y^{(0)}_{k,t}, \boldsymbol{t}^k)\, dp^k_t$$

- analytical solution for integral: $C^k_t \mid n^{(0)}_{k,t}, y^{(0)}_{k,t}, \boldsymbol{t}^k \sim \text{Beta-binomial}$

System Reliability Bounds

- Bounds for $R_{\text{sys}}\Big(t \,\Big|\, \big\{n^{(0)}_{k,t}, y^{(0)}_{k,t}, \boldsymbol{t}^k\big\}^{k=1:K}\Big)$ over 's:
  - $\min R_{\text{sys}}(\cdot)$ by $y^{(0)}_{k,t} = \underline{y}^{(0)}_{k,t}$ for any $n^{(0)}_{k,t}$ (Walter, Aslett, and Coolen 2017, Theorem 1)

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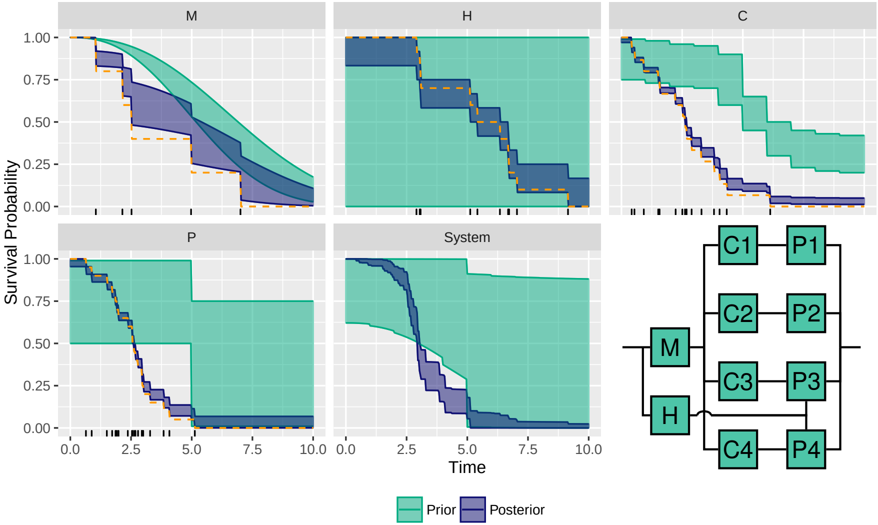
  - implemented in **R** package `ReliabilityTheory` (Aslett 2016)

System Reliability Bounds



Summary & Outlook

Summary:

- Nonparametric modeling of component reliability curves
- Bayesian model combining expert knowledge and test data
- Set of system reliability functions reflects uncertainties from limited data, vague expert information, and prior-data conflict
- Easy-to-use implementation in **R** package `ReliabilityTheory` (Aslett 2016) with the function `nonParBayesSystemInferencePriorSets()`

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Next steps:

- Allow right-censored observations (component monitoring)
- Allow dependence between components (common-cause failure, ...)
- Use for system design (where to put extra redundancy?)
- Use for maintenance planning

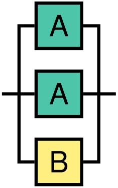
Condition-Based Maintenance Policies for Complex Systems using Component Status Montoring

Gero Walter, Simme Douwe Flapper

Eindhoven University of Technology, Eindhoven, NL
g.m.walter@tue.nl

Hannover 2016-12-08

Dynamic & Adaptive Residual Life Distributions



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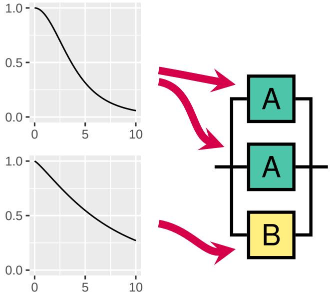

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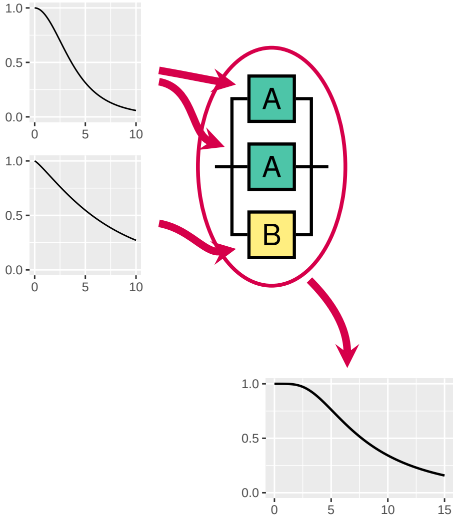

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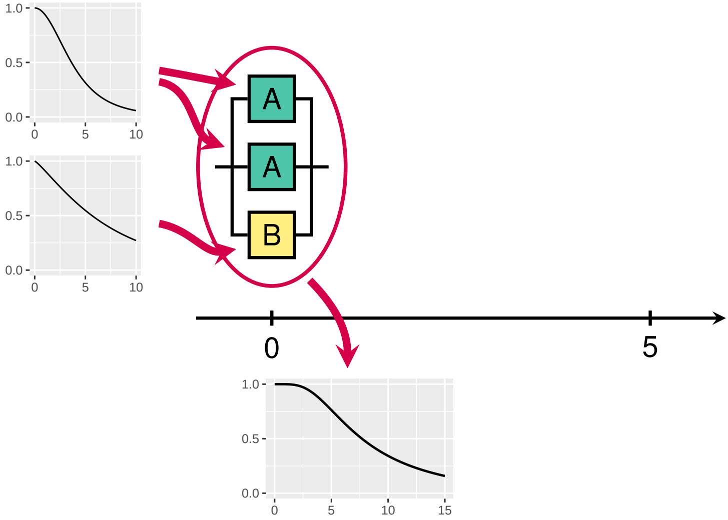

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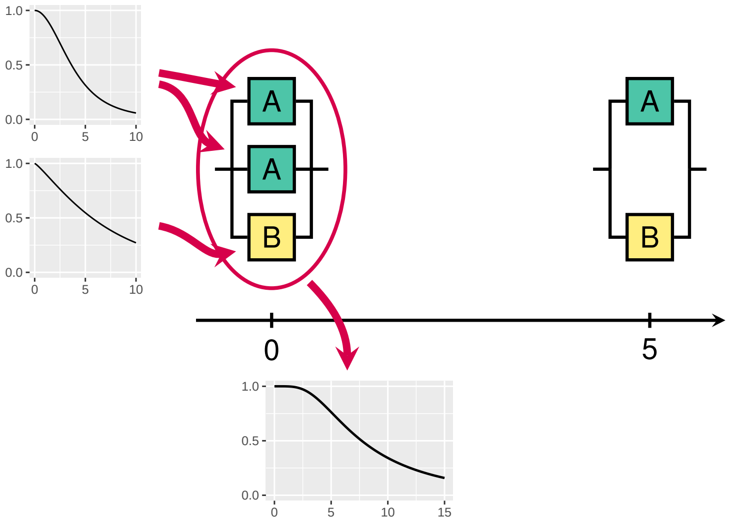

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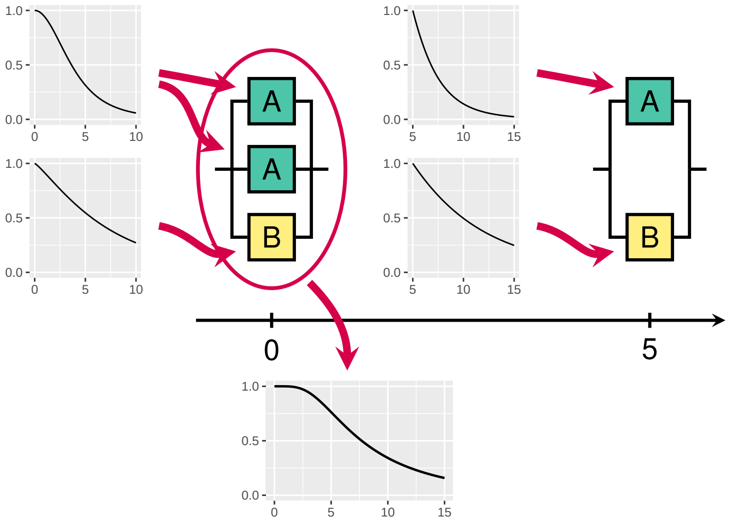

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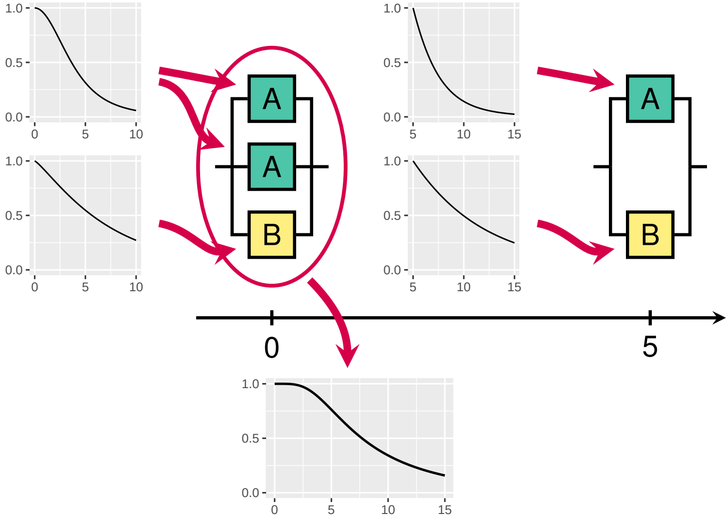

- component ageing

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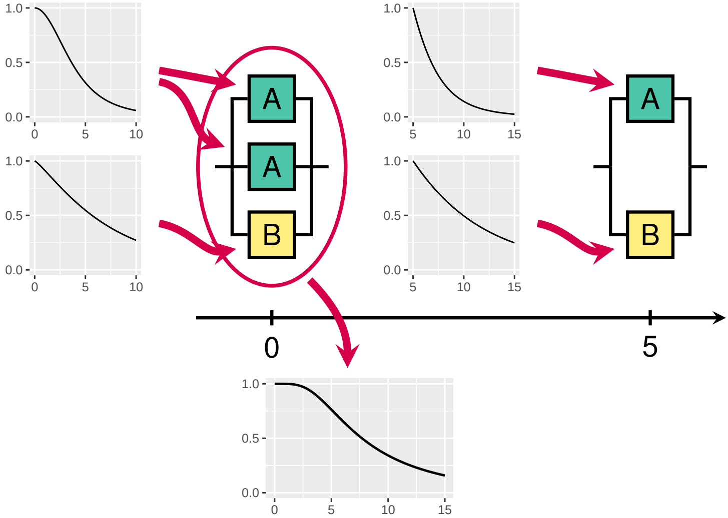

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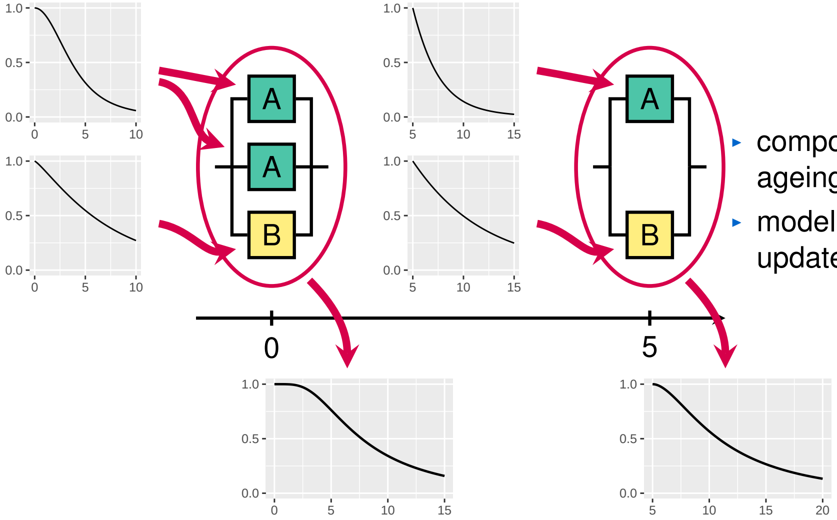

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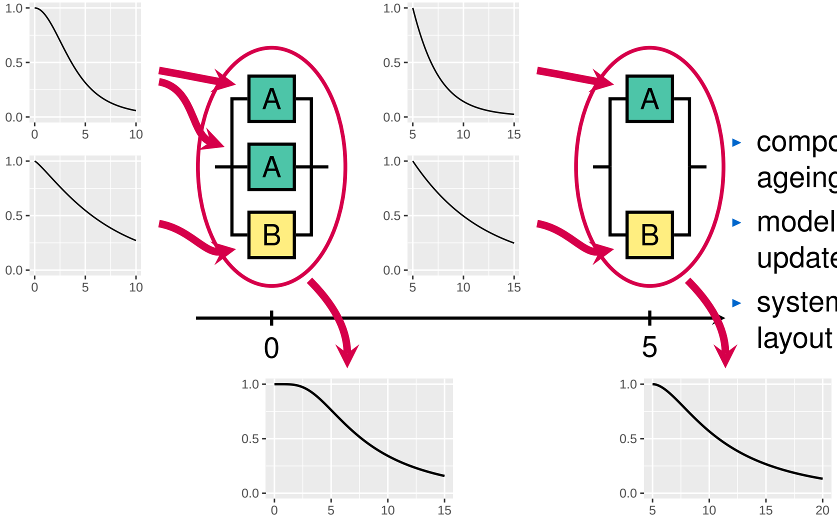

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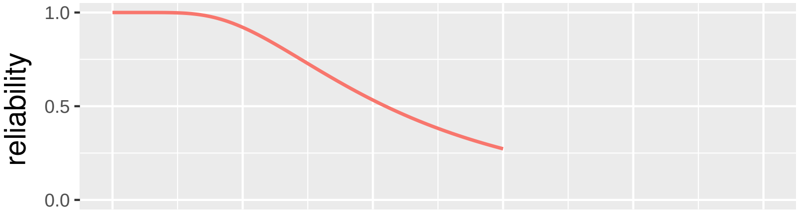
Dynamic & Adaptive Maintenance Policy



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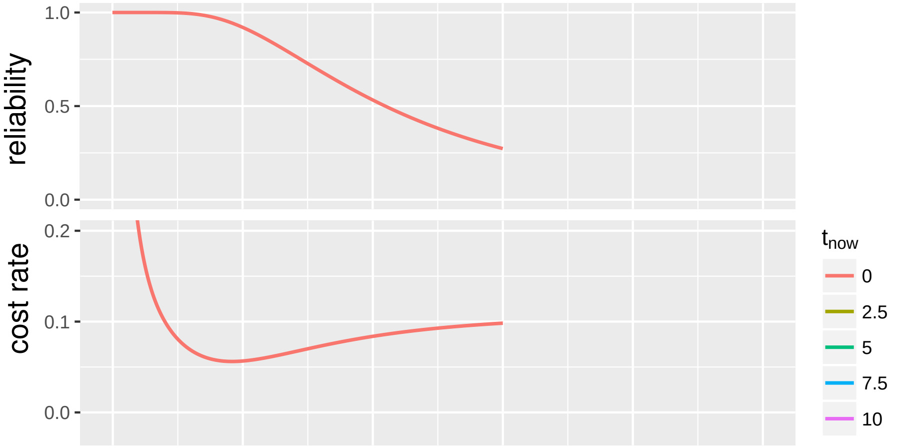

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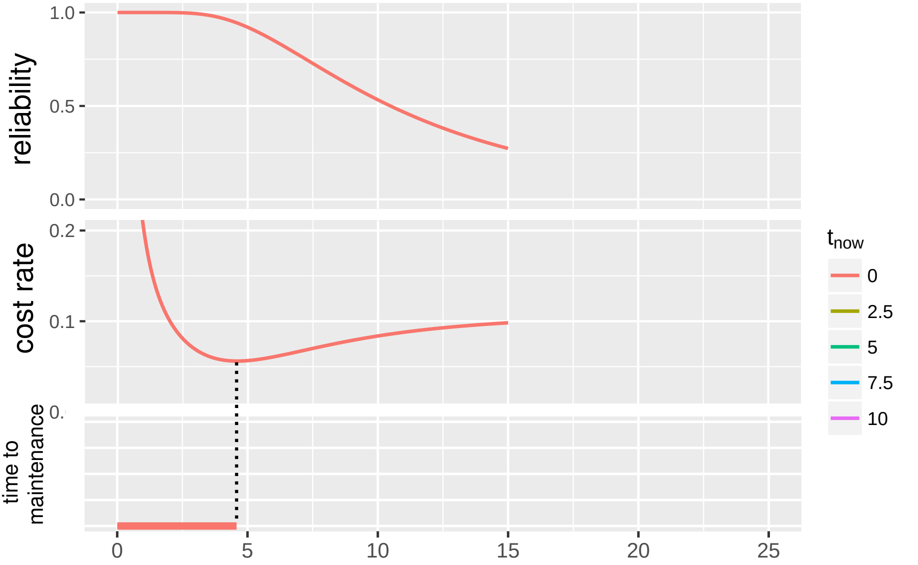

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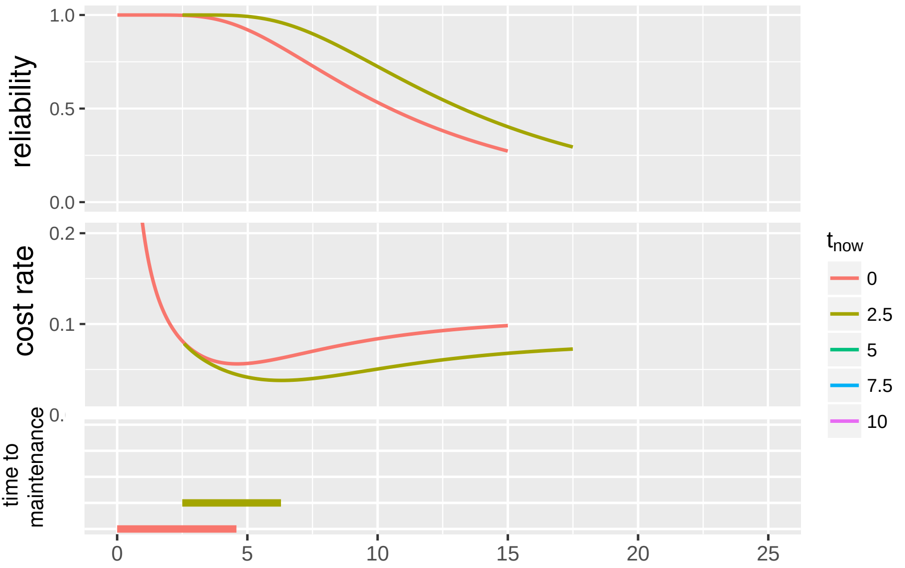

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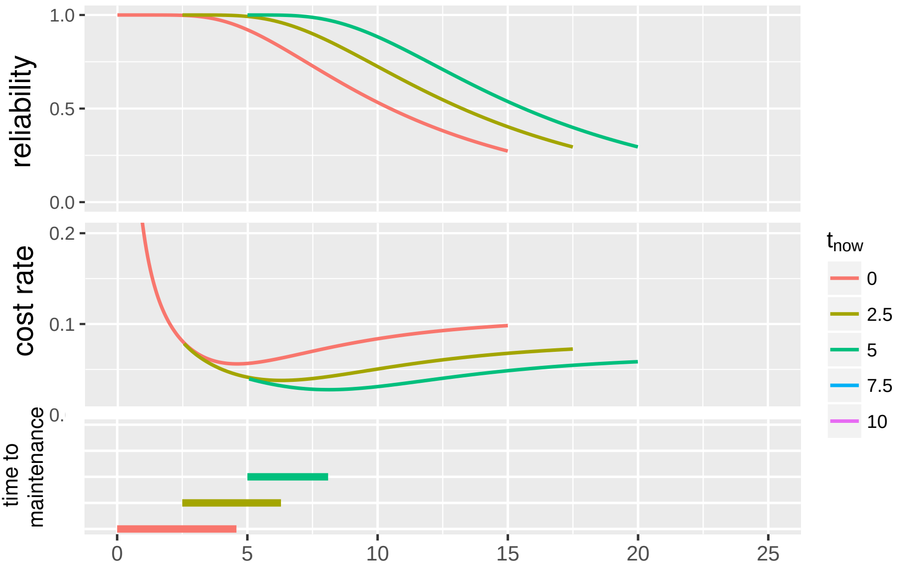

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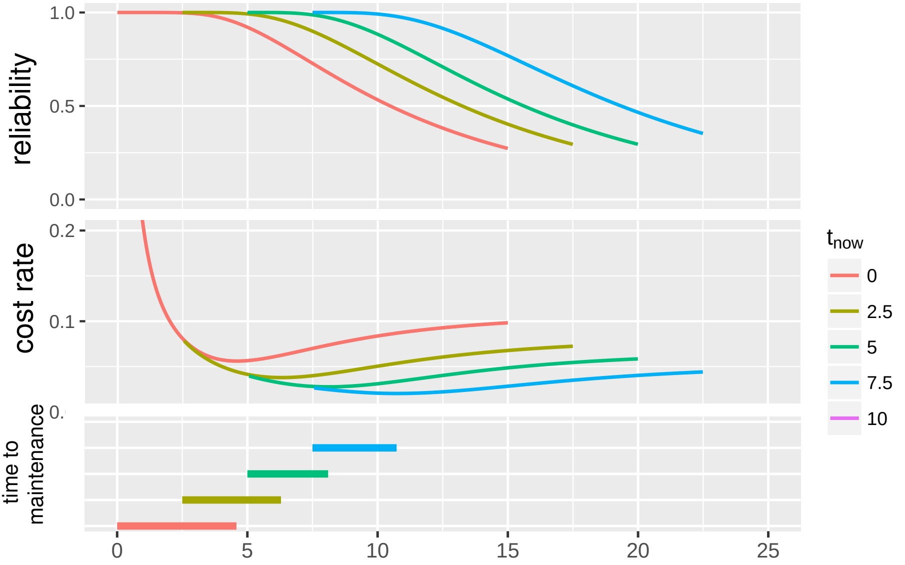

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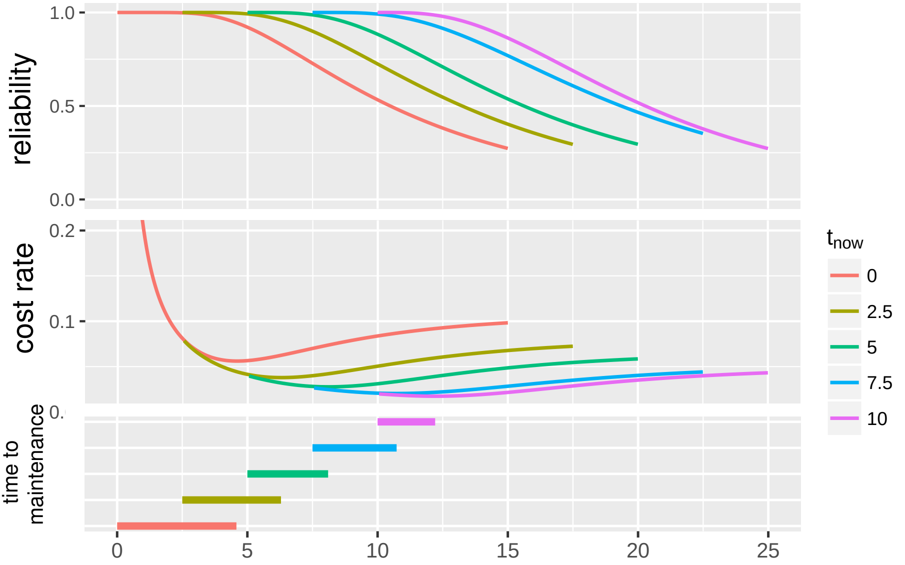

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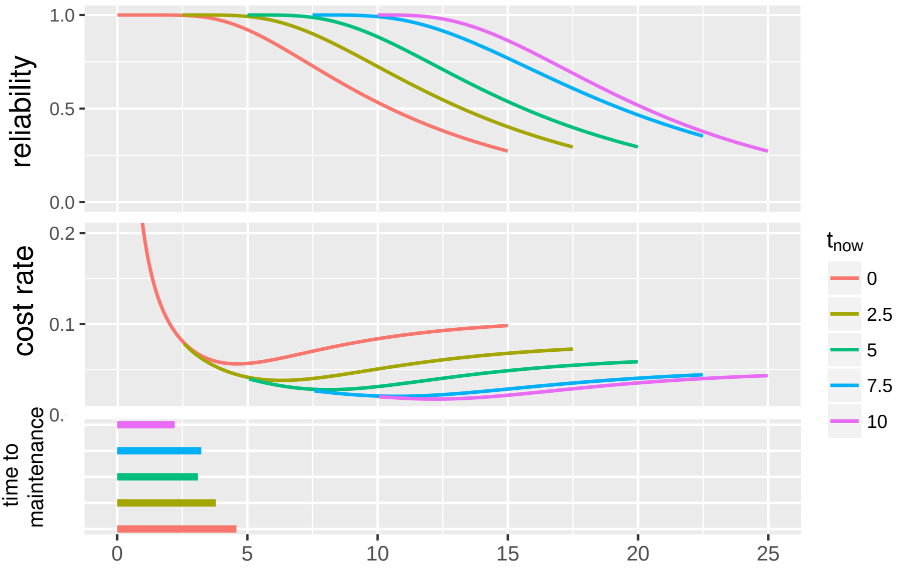

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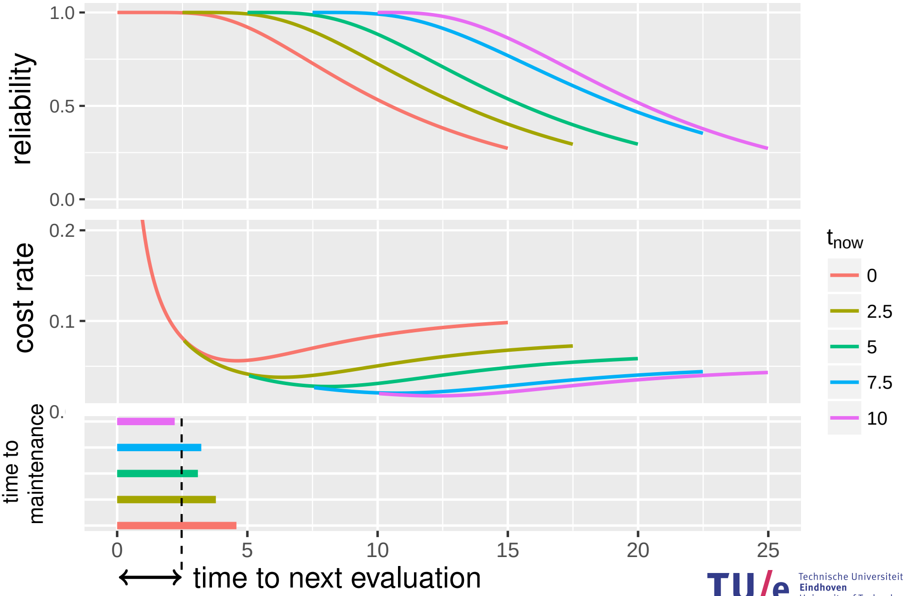
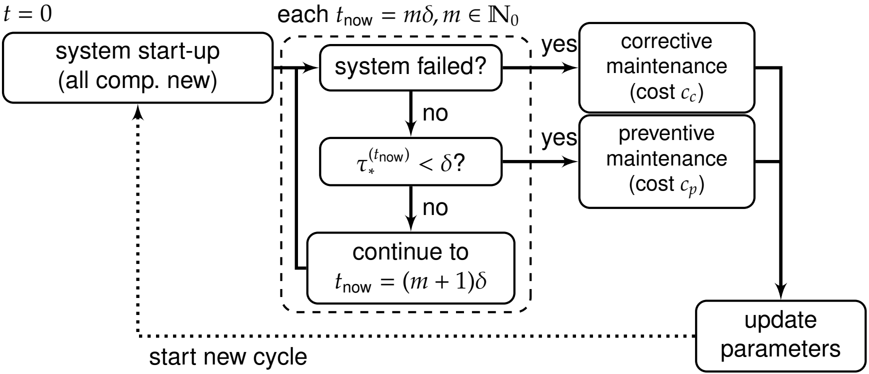


Operational Procedure



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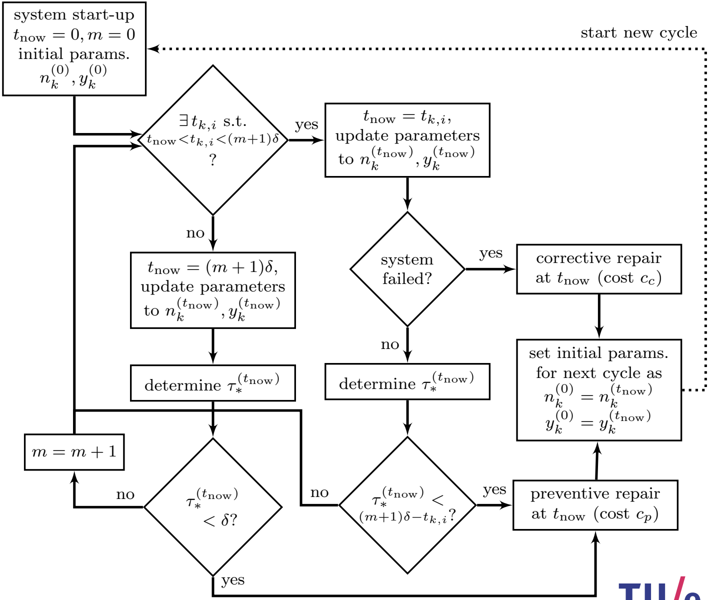

System Reliability using the Survival Signature

$T_{\text{sys}}^{(t_{\text{now}})}$ (random) time of system failure given all info. at time t_{now}

$R_{\text{sys}}^{(t_{\text{now}})}(t)$ corresponding reliability function

$c_k^{(t_{\text{now}})}$ number of type k components functioning at time t_{now}

K number of component types

$$R_{\text{sys}}^{(t_{\text{now}})}(t) = \sum_{l_1=0}^{c_1^{(t_{\text{now}})}} \cdots \sum_{l_K=0}^{c_K^{(t_{\text{now}})}} \Phi^{(t_{\text{now}})}(l_1, \ldots, l_K) \prod_{k=1}^{K} P(C_t^k = l_k \mid n_k^{(0)}, y_k^{(0)}, \boldsymbol{t}_k^{(t_{\text{now}})})$$

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Probability that l_k of the $c_k^{(t_{\text{now}})}$ k 's function

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Expected Operational Cyle Cost Rate

- τ decision variable (when to do maintenance?)
- $T_{\text{sys}}^{(t_{\text{now}})}$ (random) time of system failure, with density $f_{\text{sys}}^{(t_{\text{now}})}(t)$ and reliability function $R_{\text{sys}}^{(t_{\text{now}})}(t)$
- c_p cost of preventive maintenance action
- c_c cost of corrective maintenance action

$$g(\tau \mid T_{\text{sys}}^{(t_{\text{now}})} = t_{\text{now}} + t) = \begin{cases} c_c/(t_{\text{now}} + t) & \text{if } t < \tau \quad \text{(failure before } \tau\text{)} \\ c_p/(t_{\text{now}} + \tau) & \text{if } t \geq \tau \quad \text{(failure after } \tau\text{)} \end{cases}$$

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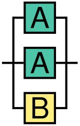
Input & Output

Inputs before start-up:

- system reliability block diagram



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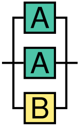
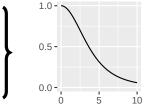
- system reliability block diagram 

- for each component type:
  - Weibull shape parameter & MTTF from expert
  - expert confidence (how sure about MTTF)
  - optional: test data



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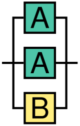

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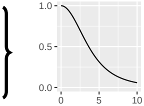
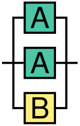


- cost parameters c_p and c_c

Input & Output



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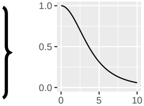

- cost parameters c_p and c_c

Input during run-time (monitoring):

- which components still work and which not

Input & Output

Inputs before start-up:

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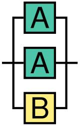

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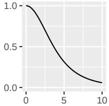

- cost parameters c_p and c_c

Input during run-time (monitoring):

- which components still work and which not

Output:

- for any time during run-time:
  cost-optimal moment to repair the system (dynamic & adaptive)

Summary & Outlook

Summary:

- Condition-based maintenance policy for complex system using only component status
- Weibull component models with known shape parameter & conjugate inverse Gamma prior for scale parameter (no MC or numerical integration for parameter update)
- minimizes expected cycle cost rate per unit time

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Summary & Outlook

Summary:

- Condition-based maintenance policy for complex system using only component status
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- minimizes expected cycle cost rate per unit time

Outlook:

- estimate / update also shape parameter (no conjugate prior!)
- interval-censored failure times, common-cause failures
- selective component replacement policy
- sets of inverse Gamma priors / nonparametric component model
  $\rightarrow$ leads to set of $R_{\text{sys}}^{(t_{\text{now}})}(t)$ and set of $g^{(t_{\text{now}})}(t)$
  $\rightarrow$ what is $\tau_*^{(t_{\text{now}})}$ then? (needs IP decision criteria)

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